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Decomposition of CI and ENSO signals using the wavelet-Based technique

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Abstract

The study is focused on six solar cycles that are studied in a comparison of related ENSO signals by using the wavelet technique. The wavelet technique is a advanced method used to analyze and understand the dynamical behavior of signals in any dimension. In this work, Discrete wavelet transformation (DWT) is applied on sun and ENSO data signals. Using the Daubechies wavelet (db 2), the Solar and ENSO parameters are decomposed into their approximation (a) and detail (d) components. By this technique, dynamical changes in both studied parameters are found effectively. The variation of solar energy was observed at a different level detailed coefficient frequencies up to 5 levels. The decomposition at level 5 shows the original signal and lower variability. Further initial and middle phase solar variation was found in the change of ENSO variability. The variability of solar energy in contrast to climate signals will be helpful in the future to understand the dynamical signal process by wavelet transformation.

Keywords: CI, ENSO, wavelet transformation

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1. INTRODUCTION

Solar energy is an important phenomenon to analysis different types of climates. The climate prevailing effects can be determined by the actual amount of solar radiation reaching a particular location. Our study uses wavelet-based support to understand the combined variation of solar-climate signals. The dynamics of solar and climate can be characterized and explore signal noise by wavelet technique. The depth of signal processing was explained with the help of the wavelet in 1984 (Grossman Morlet, 1984). Wavelet transform (WT) allows the analysis of temporal variability at different scales without requiring the data to be stationary, making it distinct from traditional spectral analysis in its ability to measure power variation. Wavelet transform (WT) is widely used in different areas such as indicator and image processing, geophysics, analysis of wave growth with breaking, the turbulent flow structures study, ocean wave dispersion, and characterization of stream flow (Farge, 1992; Graps, 1995; Torrence and Compo, 1998). Wavelet transform (WT) provides a multi-resolution representation of signals and images by breaking them into components at different scales, allowing analysis of both coarse and fine details (Mallat, 1989; Daubechies, 1998). Wavelet transform (WT) effectively detects abrupt changes in images and helps preserve and represent significant points with high accuracy. (Gkana et al., 2016). Wavelet transform (WT) is highly effective for image filtering, as it can detect edges, reduce noise, preserve small-scale details, and maintain sharp edges.

In the current study, the solar cycles were analyzed using the db wavelet transform method. The sun is a variable star whose magnetic field drives major fluctuations, influencing all solar activities occurring within the sun and its atmosphere. The sun is a variable star that undergoes significant changes due to a magnetic field, which influences all solar activities within and outside the sun's atmosphere. (Bray et al., 2006). The coronal index (CI) cycle represents solar activity in the sun's outer atmosphere. The study leverages the high variability of natural phenomena, including the decomposition of the sun and ENSO. The primary objective of decomposition is to minimize the number of temporary fluctuations in the series. The research includes conduction of wavelet transform (WT) analysis on the monthly ENSO 3.4 index and the CI solar activity to compute the comparative scaling spectrum, with their energies represented in the WT analysis as feature scales.

2. METHODOLOGY

The solar activity monthly data (CI) for the ENSO 3.4 region covering the period 1950-2016, including six cycles are used for analysis. Cycles 19, 20, 21, 22, 23 and 24 are used and similar ranges are utilized to distribute associated ENSO ranges. The online link of the applied monthly dataset can be seen at

https://www.ngdc.noaa.gov/stp/solar/corona.html

http://www.cpc.noaa.gov/products/analysis_monitoring/ensostuff/ONI_change.shtm1

Further, the various jumps and shifts through DWT are described in the next sections.

2.1. DISCRETE WAVELET TRANSFORM OF TIME SERIES

Many authentic world signals have been applied by WT. The benefit of this technique understands the signal localized discontinuities and their sharp changes (Quiroz et al., 2011). This method is also capable of converting series data that contain non-stationary power distributed across different frequency components (Daubechies, 1998). WT decomposes time-series data using wavelet windows sizes, which expand and contact at varying scales and resolutions (Santos et al., 2001). Furthermore, the decomposition of the one-dimensional signal can be separated into 2-dimensional times of time-frequency domains at the equal duration of time (Adamowski et al., 2009). The two commonly used types of wavelets are the Discrete Wavelet Transform (DWT) and the CWT (Chou, 2007). Discrete Wavelet Transform (DWT) has been employed in this research work, which is particularly useful for identifying sharp changes or discontinuities in time series data. (Partal and Kucuk, 2006). The Discrete Wavelet Transform (DWT) uses scale and translation parameters that change in power powers of two, known as dyadic dilation. These scaling and translation parameters are discretized and can be expressed as follows. (Fugal, 2009; Svalgaard, 2010).

$$DWT_x(m,n) = 2^{-m/2} \sum_{t=0}^{T-1} x(t)\, \Psi\left(\frac{t-n.2^m}{2^m}\right) \qquad (1)$$

The duration(length) of the signal x(t) is denoted by 'T'.
m and n are integer variables representing the scaling and translation parameters, respectively.

2.2. TIME SERIES DECOMPOSITION USING WAVELET (DWT)

DWT produces a newly formed vector with the same dimensional size as the original vector (Chou, 2011). The DWT approach is based on filters or multi-resolution analysis (MRA) that implements decomposition and reconstruction (Catalao et al., 2010 and Mallet, 1989 & 1999). In our paper decomposition using 5 level decomposition for CI cycles and associated ENSO data. The original CI and ENSO signals, 5 details (D) and 1 approximation (a), are studied using Daubechies wavelets (db2). This wavelet provides compact support (nonzero function) on a finite interval, i.e., full-scale orthonormality and transformation properties field for localization of events, these functions are applicable in dependent on the time of the signals. (Vonesch et al., 2007; Popivanov and Miller, 2002).

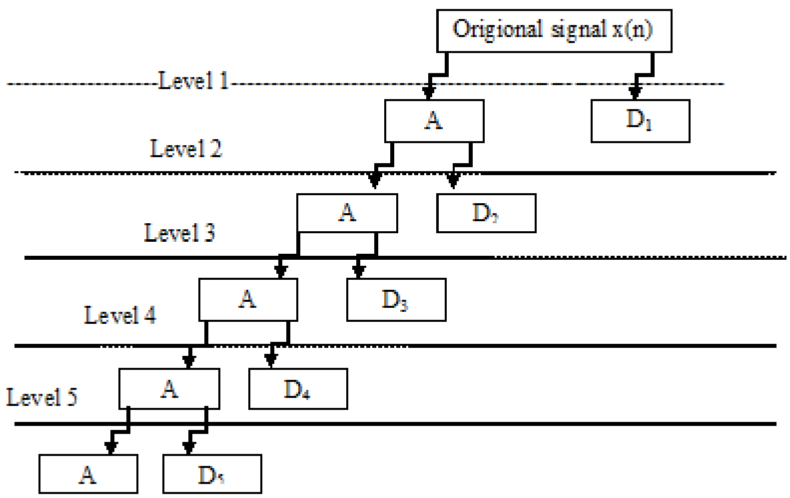
The decomposition expressed as a finite sum of wavelets across scales or resolution levels, suppose time series x(n) as

$$x(n) = \sum_j \sum_k c_{jk}\, \Psi(2^j nT - k) \qquad (2)$$

Here c_{jk} denotes a set of coefficients, while $\Psi(2^j nT - k)$ represents the wavelet function at the j-th scale level that is shifted by k samples (Daubechies, 1998; Mallet, 1999 and Al-Alawi et al., 2008).

In the transformation phase, the original signal is decomposed into detailed coefficients Dj(k) at various levels (j=1,2,.....,j) and residual approximation signal Aj(k). The Mallet pyramid algorithm filtering confines the high pass and low pass process of a signal (Mallet, 1999). Further decomposes signal can be re-created in the form of original resolution time-series x(n). The re-creation of obtained detail coefficient Dj(k) and residual signal Aj(k) joint together by given expression and their flow of process given below

$$S=x(n) = D_1(n) + D_2(n) + \cdots + D_J\,(n) + A_J\,(n) \tag{3}$$



The Fourier transform is a widely used technique for achieving time and frequency localization by using a specific window size, effectively addressing the issue of frequency localization. While we could have repeated the analysis with various fixed window sizes, using a fixed window width presented several limitations. Moreover, if vibrations occur within a low-frequency window, frequency localization information is lost; similarly, with high-frequency vibrations, time localization is compromised. Wavelet Transform (WT) addresses this issue by decomposing a one-dimensional time series and simultaneously converting it into two diffusion time series. Additionally, Wavelet Transform (WT) enables us to extract information about periodic signals within a time series and observe how the time series evolves over time (Santos et al., 2001).

3. RESULTS AND DISCUSSION

Wavelet analyses are widely applied to study fluctuations in solar activity and climatic signals. That is the reason we applied to WT for solar and climate signals in our work. The Coronal Index (CI) reflects long-term variations in solar activity within the solar corona. Discrete Wavelet Transform (DWT) was applied to the CI and ENSO time series signals using high-pass and low-pass filtering techniques. Daubechies wavelets (db2) are used to decompose signal (s) into approximation and detail coefficients of the original signal (S). The db wavelet exhibits a self-similar pattern and is known for its excellent smoothness and strong localization properties. Using the db wavelet, a signal is decomposed into two mutually orthonormal components: high –frequency detail part and a low-frequency approximation(residual) part. Both components were retained, where the approximation represented the long-term behavior of the data and closely resembled the original time series. whereas the detail component represented th short-term variations within the defined range. The decomposition analysis includes six Coronal Index (CI) cycles together with their associated ENSO cycles. The creation of figures 1-12 is based on the relationship between amplitude and time series data. The first panel displays the variability signal (S) of the CI-ENSO time series, while the second panel presents the adjustment coefficient "a5."

The fitted coefficient is utilized to identify short-term variations from the peaks of long-term variations. The other panels, d1, d2, d3, d4, and d5, represent the detailed coefficients of the CI and ENSO time series. These details provide insights into energy across various frequency scales. CI refers to solar activity related to solar energy variations, whereas ENSO is associated with anomalies in sea temperature.

In the wavelet decomposition results, oscillatory behavior was detected in both the CI and ENSO phases. The high-frequency components consist of the detailed coefficients of the CI cycle observed during the early and primary stage of the temporal evolution. The coefficients in the CI details also indicate the field's strength, oscillating between positive and negative values. This variation indicates a relationship with an intense and highly variable solar magnetic field on the Sun. The ENSO detail coefficient corresponds to the warm and cold phases of sea surface temperature variability. It was found that during the peak or dominant phase of the corresponding cycles, observed that during the maximum (or main phase) of relevant cycles, CI cycles exhibited high frequency and strong amplitude, indicating increased stability for high-level decomposition. Additionally, apart from the initial and main phases, the middle and starting phases do not show consistently strong fluctuations. In contrast, ENSO displays more significant fluctuations in the initial phase, with a typical initial minimum in the CI cycle and a more active main phase in ENSO propagation. Furthermore, the approximation trend was identified as the signal's slowest component. The subsequent approximation, "a5," in our analysis, contains less high-frequency information. Wavelet Transform (WT) is valuable for identifying patterns in signals. It is important to note that abrupt changes in the signal cause the approximation to gradually diverge from the real time series. In our analysis, the a5 component exhibited a good level of similarity with the original signal. The following figures illustrate the breakdown of the ENSO and CI signals.

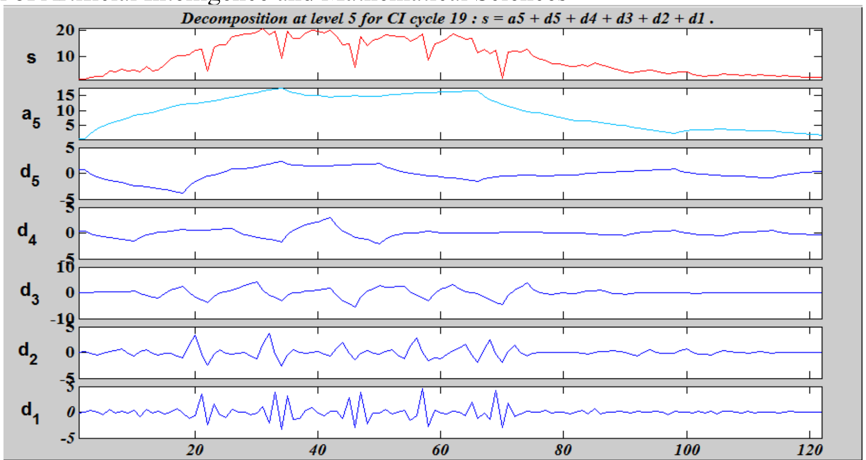


Fig. 1 The time series S= x(n) of CI cycle 19 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

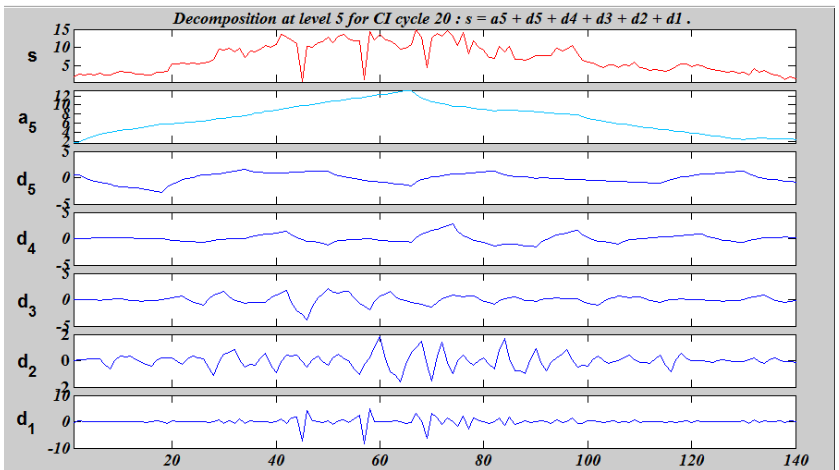


Fig. 2 The time series S= x(n) of CI cycle 20 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

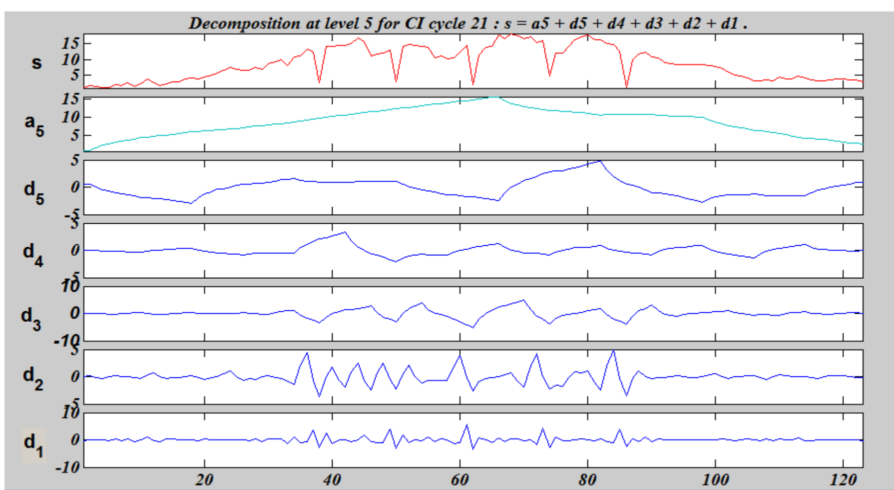


Fig. 3 The time series S= x(n) of CI cycle 21 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

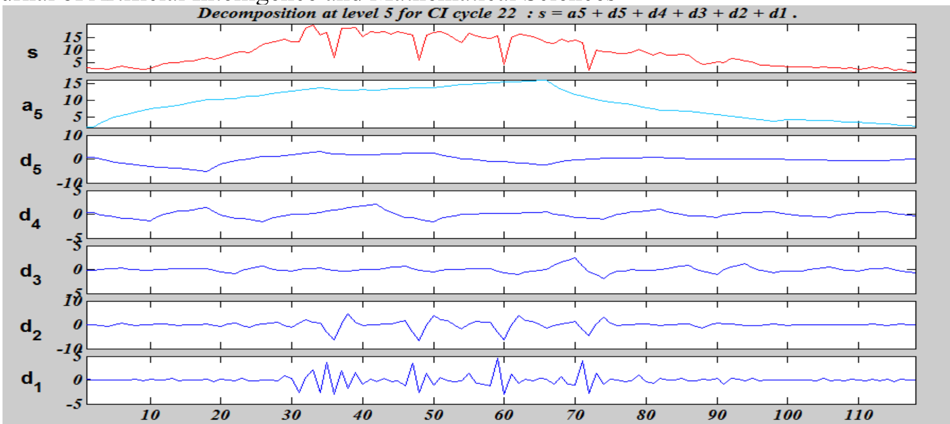


Fig. 4 The time series S= x(n) of CI cycle 22 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

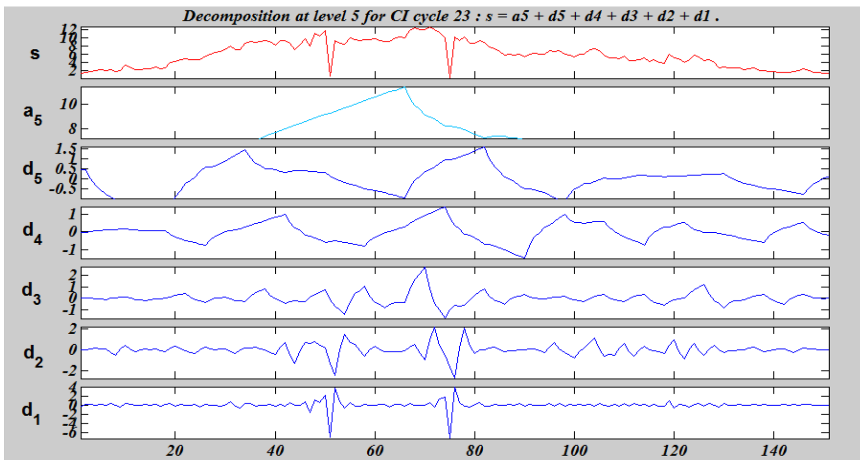


Fig. 5 The time series S= x(n) of CI cycle 23 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

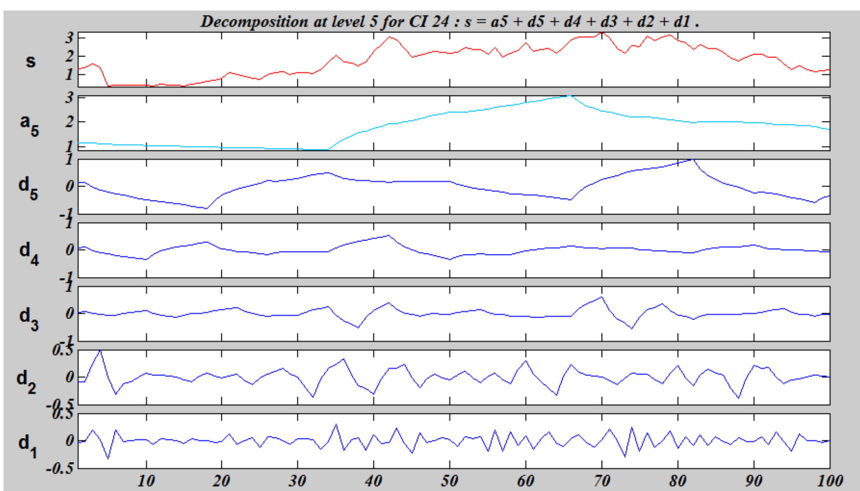


Fig. 6 The time series S= x(n) of CI cycle 24 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

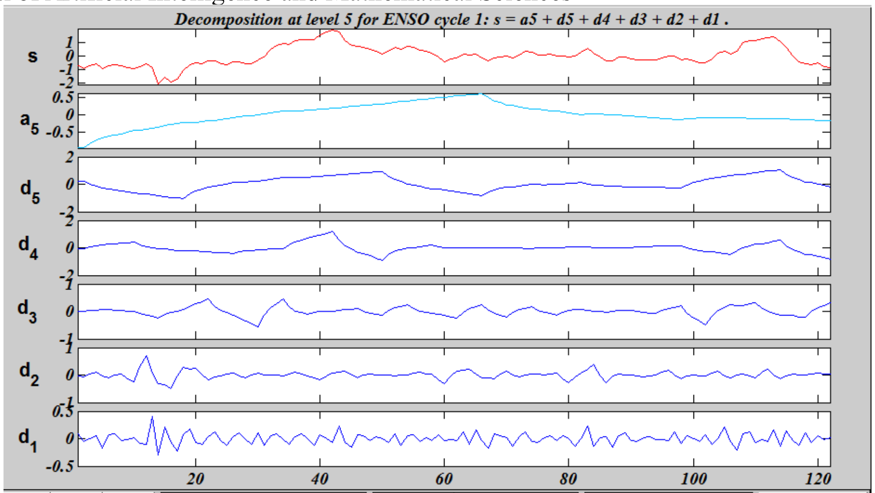


Fig. 7 The time series S= x(n) of ENSO cycle 19 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

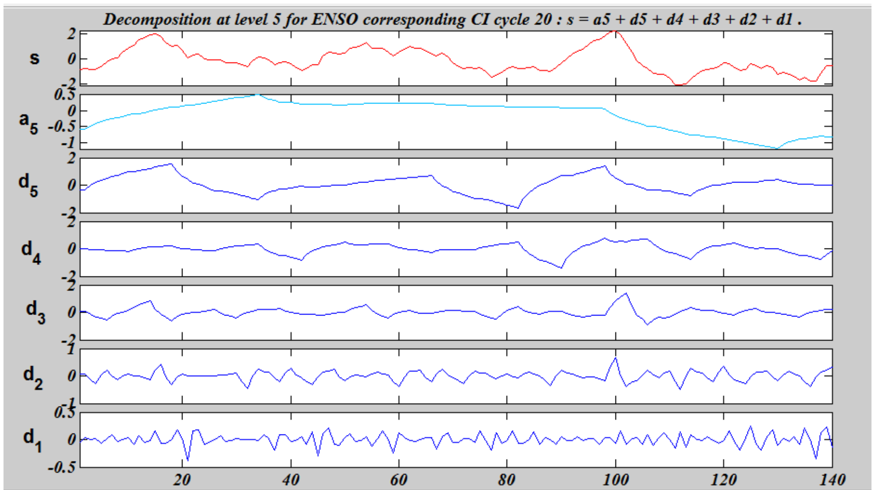


Fig. 8 The time series S= x(n) of ENSO cycle 20 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

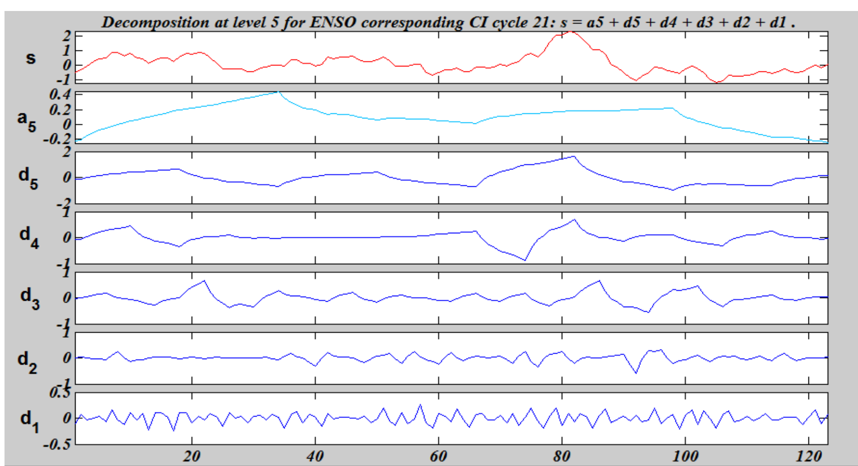


Fig. 9 The time series S= x(n) of ENSO cycle 21 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

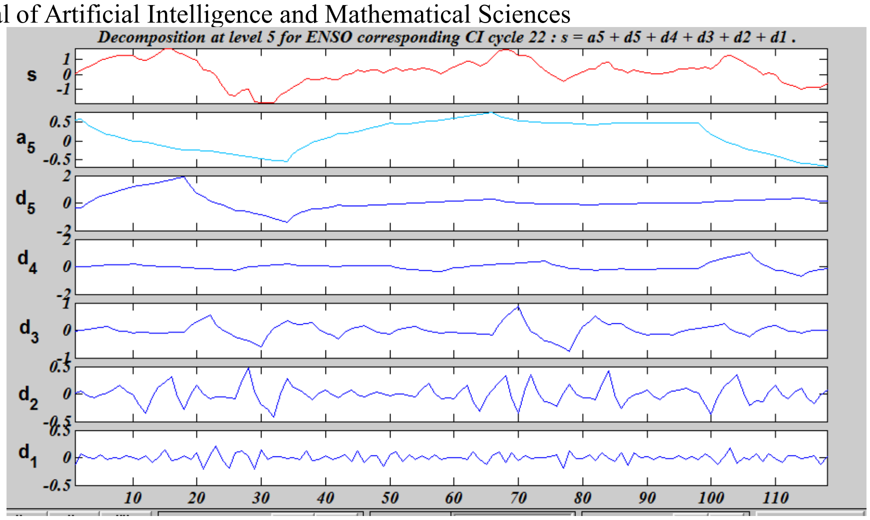


Fig. 10 The time series S= x(n) of ENSO cycle 22 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

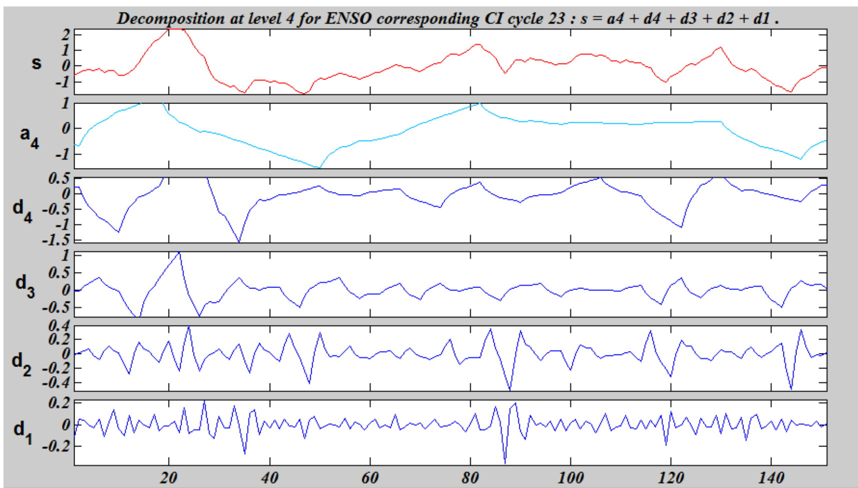


Fig. 11 The time series S= x(n) of ENSO cycle 23 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

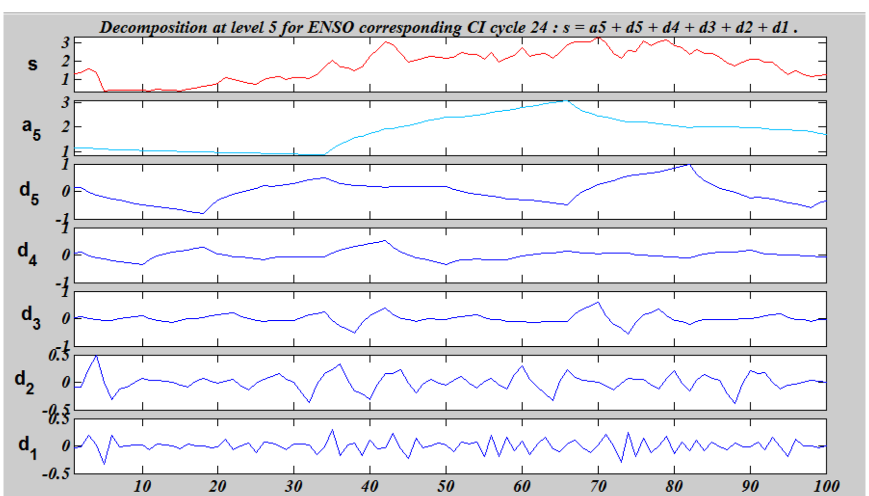


Fig. 12 The time series S= x(n) of ENSO cycle 24 measured wavelet decomposition (db 2), signal x(n) is decomposed at 5th level into detailed coefficients d1 to d5 and a coarse approximation component a5

All CI and ENSO signals are represented from the first to fifth detail levels (d1-d5), along with the fifth-level coarse approximation (a5), are shown at the same resolution as the signal x(n). The variability of the signal's changes notably across different levels. As the wavelet level increases, the variation in coefficients decreases, making their prediction easier.

4. CONCLUSION

Wavelet transformation is a multiscale technique that provides enhanced insight into high peaks and series noise. The Discrete Wavelet Transform (DWT) is useful for highlighting and distinguishing the underlying phenomenological differences of solar cycles-ENSO data. Fluctuations in solar data are noticeable at the start and end of CI cycles. Likewise, the ENSO cycle exhibits a relationship between its primary phase and the subsequent recovery phase. Similarly, the ENSO cycle also demonstrates a linkage between the main and recovery phases. The study's enhanced detail coefficient at level 5 and the level 5 approximation exhibits a reasonable representation of the actual signal trends for both ENSO and CI cycles. Thus, it can be concluded that the WT approach is effective in measuring variations in solar and climate dynamics by offering insights into their local and adaptive characteristics.

References

[1]. Adamowski, K., Prokoph, A., & Adamowski, J. (2009). Development of a new method of wavelet aided trend detection and estimation. Hydrological Processes: An International Journal, 23(18), 2686-2696.

[2]. Al-Alawi, S. M., Abdul-Wahab, S. A., & Bakheit, C. S. (2008). Combining principal component regression and artificial neural networks for more accurate predictions of ground-level ozone. Environmental Modelling & Software, 23(4), 396-403.

[3]. Bray, R. J., Cram, L. E., Durrant, C., & Loughhead, R. E. (2006). Plasma loops in the solar corona (Vol. 18). Cambridge University Press.

[4]. Catalão, J. P. D. S., Pousinho, H. M. I., & Mendes, V. M. F. (2010). Hybrid wavelet-PSO-ANFIS approach for short-term electricity prices forecasting. IEEE Transactions on Power Systems, 26(1), 137-144.

[5]. Chou, C. M. (2007). Applying multi-resolution analysis to differential hydrological grey models with dual series. Journal of Hydrology, 332(1-2), 174-186.

[6]. Chou, C. M. (2011). Wavelet-based multi-scale entropy analysis of complex rainfall time series. Entropy, 13(1), 241-253.

[7]. Daubechies, I. (1998) Orthonormal Bases of Compactly Supported Wavelets. Communications on Pure and Applied Mathematics , 41, 909-996. https://doi.org/10.1002/cpa.3160410705.

[8]. Farge, M. (1992) Wavelet Transforms and Their Applications to Turbulence. A nnual Review of Fluid Mechanics , 24, 395-457. https://doi.org/10.1146/annurev.fl.24.010192.002143.

[9]. Fugal, D. L. (2009). Conceptual wavelets in digital signal processing: an in-depth, practical approach for the non-mathematician. Space & Signals Technical Pub.

[10]. Gkana, A., & Zachilas, L. (2016). Re-evaluation of predictive models in light of new data: Sunspot number version 2.0. Solar Physics, 291(8), 2457-2472.

[11]. Graps, A. (1995) An Introduction to Wavelets. IEEE Computational Science and Engineering , 2, 50-61. https://doi.org/10.1109/99.388960.

[12]. Grossman, A. and Morlet, J. (1984) Decomposition of Hardy Functions into Square Integrable Wavelets of Constant Shape. SIAM Journal on Mathematical Analysis , 15, 723-736. https://doi.org/10.1137/0515056.

[13]. IPCC, Technical Paper V.2002: Climate change and Biodiversity.

[14]. Mallat, S. (1999). A wavelet tour of signal processing. Elsevier.

[15]. Mallat, S.G. (1989) A Theory for Multiresolution Signal Decomposition the Wavelet.

[16]. Partal, T., & Küçük, M. (2006). Long-term trend analysis using discrete wavelet components of annual precipitations measurements in Marmara region (Turkey). Physics and Chemistry of the Earth, Parts A/B/C, 31(18), 1189-1200.

[17]. Popivanov, I., & Miller, R. J. (2002, February). Similarity search over time-series data using wavelets. In Proceedings 18th international conference on data engineering (pp. 212-221). IEEE.

[18]. Quiroz, R., Yarlequé, C., Posadas, A., Mares, V., & Immerzeel, W. W. (2011). Improving daily rainfall estimation from NDVI using a wavelet transform. Environmental Modelling & Software, 26(2), 201-209.

[19]. Santos, C.A.G., Galvao, C.O., Suzuki, K. and Trigo, R.M. (2001) Matsuyama City Rainfall Data Analysis using Wavelet Transform. Annual Journal of Hydraulic E ngineering , 45, 211-216.

[20]. Svalgaard L, Updating the historical sunspot record, in SOHO-23:Understanding a peculiar solar minimum.Proceedings of a workshop held 21–25September in Northeast Harbor, Maine,USA, ed. by SR Cranmer, JT Hoeksema,JL Kohl. ASP Conference Series,(Astronomical Society of the Pacific, San Francisco, 2010). 2009;428:297.

[21]. Torrence, C. and Compo, G.P. (1998) A Practical Guide to Wavelet Analysis. Bull etin of the American Meteorological Society , 79, 61-78. https://doi.org/10.1175/1520-0477(1998)079<0061:APGTWA>2.0.CO;2

[22]. Vonesch, C., Blu, T., & Unser, M. (2007). Generalized Daubechies wavelet families. IEEE Transactions on Signal Processing, 55(9), 4415-4429.

[23]. Wang, B., W. Renguang and K. M. Lan, (2001): Interannual Variability of the Asian Summer Monsoon: Contrasts between the Indian and the Western North Pacific–East Asian Monsoons. Journal of Climate, (14), 4073-4090.