

## **Evaluating the Efficiency of Schools in Gilgit-Baltistan: An Exploratory Study**

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**Abstract:** It is known that a well-established education system of a country contributes positively to both the citizens of a nation and its overall economic development and growth. The objective of the study is to identify efficiency and faults in the education sector of Gilgit-Baltistan. For this purpose, this study intent to estimates and evaluates the performance of 1149 government girls and boys' schools in seven districts of Gilgit-Baltistan using data envelopment analysis (DEA) which helps to highlight ways to reduce inputs or increase outputs necessary for insufficient schools. The utilized data was taken from Directorate of Education Gilgit-Baltistan for the year 2012 to 2013. The results indicate that while the majority of government schools in the Gigit-Baltistan are efficient, a number of them lack in performance due to inadequate use of the available resources. Furthermore, girl's schools are more efficient as compared to boys' schools in the studied region. In short, this sort of research will be helpful in improving best performance in educational sectors in both government and private sectors.

**Keywords:** Data Envelopment Analysis (DEA), Efficiency analysis, Schools, Decision Making Units (DMUs)

### **1. Introduction**

This research intends to present an overview of the Government Schoolings of Gilgit-Baltistan. Clear analysis of the performance of education in Gilgit-Baltistan is slowed down by the lack of availability of data. On the other hand, the significance of education is well recognized for individual achievement as well as for decision-making affluence. But the question arise as to what an excellent school is and how the competence of a school can be recovered, which are always of attention for parents, teachers, educationists, and policy makers [1].

It is known that DEA is an influential linear programming mathematical technique commonly applied in the assessment of the effectiveness of Decision Making Units (DMUs) [2], these DMUs may be production units (i.e. points of sales, bank branches, dealers, and franchisees), administrative organizations, police departments, or hospitals [3]. Therefore, the DEA technique assists the assessment of relative efficiency between production units by conducting a comparative analysis of the selected decision-

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making units (DMUs). This method aims to determine the most efficient units, which are described by their capability to attain maximum productivity with the resources available to them [4-5].

DEA involves various input and yield computations and, does not require any particular assumption on the efficient association between those inputs or production (i.e., on the efficient type of the manufacturing tools). Therefore, DEA estimates the technological efficiencies that are free of probable errors resulting from incorrect assumptions. Another excellent characteristic of DEA is that the weights allocated to both input and production can differ by study [6].

Since DEA is a valuable tool for computing and recognizing the efficiency of decision-making units, it has also been used to determine the best weights by measuring efficiency. DEA aims to recognize the efficiency of a system or a decision-making unit [7]. One important advantage of the DEA model is its unique weight flexibility, which estimates maximum efficiency by DMU. DEA offers several advantages. For example, unlike multiple regression analysis, DEA does not require the estimation of a production function. Additionally, DEA allows for the estimation of the efficiency of multiple manufacturers. The intense point method of DEA compares inefficient firms to actual “best” firms while accounting for differing input arrangements or technologies [8]. Efficiency can be expressed as the ratio of the weighted summation of outputs of a DMU divided by the weighted summation of its inputs. When applied to all other DMUs, none can have efficiency greater than one [9]. The best observed components are somewhat efficient and are recognized as a DEA competence score of  $\theta = 1$ . The incompetent components are known as efficiency score  $\theta < 1$ . The efficiency rating of DEA will generally be written between zero and 1 [2]. The aim of efficiency measurements is to measure relative technical efficiency by establishing benchmark firms (efficient DMUs in the industry) and then comparing inefficient firms to them [8].

In a general context, efficiency means a procedure that facilitates the achievement of a particular aim under the most advantageous conditions, specifically at the least possible cost. It reflects the capability of an individual, a group, a machine, or a technique to yield maximum outcomes with the least amount of resources, expenses, effort, or energy. This concept reflects the proficiency in delivering effective performance in a logical manner for a particular task or role, as well as the optimization of the resources that are available or assign to achieve a desired outcome [10]. This paper is distributed in four sections, in first section; introduction and related literature described, in the section two; data source and detail explanation of methodology, and section three presented important findings and discussion. Similarly, section four concludes the key results and recommendation.

## 2. Data and Method of Analysis

The current study examines the competence differences of schools in seven districts of Gilgit-Baltistan. The Data was collected from the Directorate of Education Gilgit-Baltistan (2012-2013). There are 1149 government schools for girls and boys at the districts level.

This investigation has measured school related variables on a district-wise basis. The variables applied to examine the efficiency of government schools in Gilgit-Baltistan are as follows:

Input variables:

- i. Total number of schools,
- ii. Total number of class rooms,
- iii. Total number of teachers,

Output variable:

- I. Total number of new enrollments (2012-13).

In this manuscript, we used Data Envelopment Analysis (DEA) technique to estimate the relative effectiveness of Girls and Boys Schools in Gilgit-Baltistan. Many studies recommend a variety of models as DEA that use multiple inputs and outputs to compare the efficiency of two or more processes [2-11-12]. Efficiency (i.e. technical efficiency) is the association among inputs applied in some action and the formed productions and their optimal values [12]. Talluri (2000) [13] defined the highest competency of every DMU,s as a ratio of the overall weighted outputs to the overall weighted inputs [14] i.e.

$$\text{Efficiency} = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}}$$

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The DEA is a linear programming methodology applied to locate the set of coefficients  $u$ 's and  $v$ 's that will present the maximum efficiency relation of production (output) to inputs for the DMU's being estimated. Many researchers [2-13-15-16-17] expressed this relationship mathematically as, consider the objective function

$$\text{Maximum efficiency } \theta = \frac{\sum_{r=1}^n u_r y_{r\theta}}{\sum_{i=1}^m v_i x_{i\theta}}$$

$$\text{Maximum efficiency } \theta = \frac{u_1 y_{1\theta} + u_2 y_{2\theta} + \dots + u_r y_{r\theta}}{v_1 x_{1\theta} + v_2 x_{2\theta} + \dots + v_m x_{m\theta}}$$

From the constraint that the similar set of ' $u$ ' and ' $v$ ' coefficients concern to all other DMU's being correlated, it is stated that no DMU's will be more than 100% proficient.

$$\text{DMU1 } \frac{u_1 y_{11} + u_2 y_{21} + \dots + u_r y_{r1}}{v_1 x_{11} + v_2 x_{21} + \dots + v_m x_{m1}} = \frac{\sum_{r=1}^n u_r y_{r1}}{\sum_{i=1}^m v_i x_{i1}} \leq 1$$

$$\text{DMU2 } \frac{u_1 y_{12} + u_2 y_{22} + \dots + u_r y_{r2}}{v_1 x_{12} + v_2 x_{22} + \dots + v_m x_{m2}} = \frac{\sum_{r=1}^n u_r y_{r2}}{\sum_{i=1}^m v_i x_{i2}} \leq 1$$

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$$\text{DMUj } \frac{u_1 y_{1j} + u_2 y_{2j} + \dots + u_r y_{rj}}{v_1 x_{1j} + v_2 x_{2j} + \dots + v_m x_{mj}} = \frac{\sum_{r=1}^n u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1$$

$$u_1, u_2, \dots, u_n > 0, \text{ and } v_1, v_2, \dots, v_n > 0$$

Where the mathematical symbols indicates

$j$  = Total number of DMU's

DMUj = Decision Making Unit quantity j

$\theta$  = Effectiveness of DMU,s

$y_{rj}$  = total sum of quantity of output ' $r$ ' utilized in DMUj

$x_{ij}$  = total sum of quantity of input ' $i$ ' utilized in DMUj

$i$  = Total number of inputs

$r$  = Total number of outputs

$u_r$  = Coefficient allocate in DEA to production (output) ' $r$ '

$v_i$  = Coefficient allocated in DEA to input ' $i$ '

As a DMU obtains its maximum value of output (100%) with a certain quantity of inputs by using the above technique then it is called technically competent, i.e. it is working at the production likelihood frontier. On the contrary, if it constructs a maximum value  $< 100\%$ , then we say that the DMU will be incompetent. Numerous software such as Matlab, Stata, and Excel Solver are utilized to find the efficiency of DMUs. Excel Solver helps us to calculate the efficiency of schooling data. The Excel spreadsheet applies a program called "add-in". The "add-in" that will be facilitating for DEA is the Excel Solver [2]. Figure 1 described the flow chart of complete concept of DEA approach.

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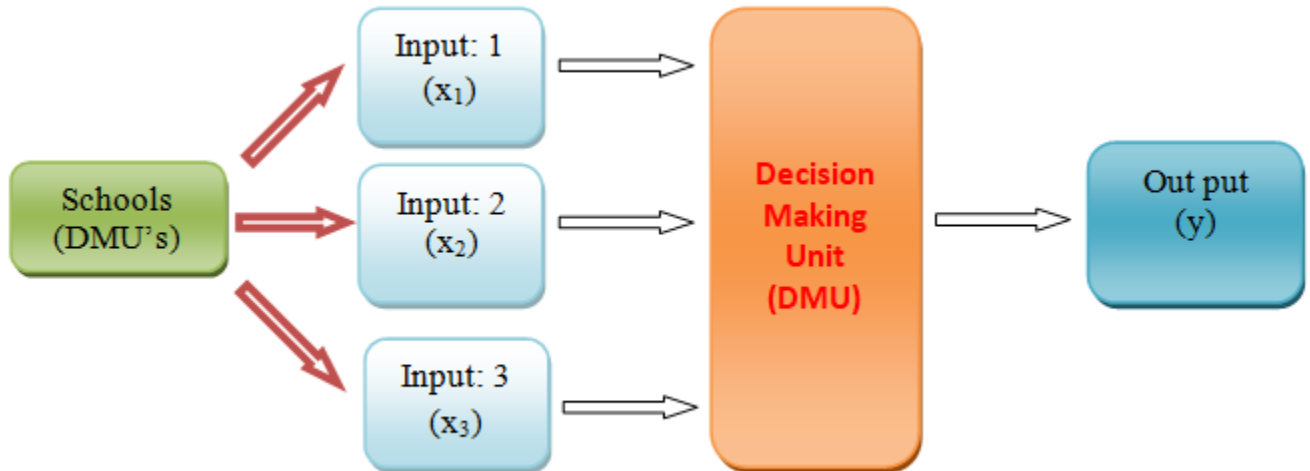


Figure 1 Flow chart for DEA concept

### 3. Results and Discussions

It is essential for every society in advancing the enhancement of education for its citizens. A well-educated population, supported by an effective educational system, is closely related to the potential for advancing economic growth [16]. This section illustrates the observed outcome of the research and measures the efficiency achieved with respective constant returns to scale (CRS). Table 1 and Table 2 compare the distribution of efficiency scores of girls and boys schools in seven districts of Gilgit-Baltistan. Table 1 show that the efficiency scores of girl's schools range from 81.16% to 100%. This indicates that Ghanche (81.16%), Astor (81.48%), and Ghizer (83.12%) are inefficient districts, while Gilgit (100%), Skardu (100%), Diamar (100%), and Hunza-Nagar (100%) are technically efficient, as shown in Figure 2 with a dark orange cell background.

In Table 2, indicate the efficiency scores of boys' schools which range from 65.81% to 100%. The efficient districts (Skardu, Diamar, and Hunza-Nagar) have a 100% efficiency score and are shown in Figure 3 with a blue cell background. The values 0.9713, 0.9661, 0.8576, and 0.6581 in the inefficient districts Astor, Gilgit, Ghizer, and Ghanche mean that their inputs can simultaneously be reduced by a factor of 1-0.9713 (i.e., 2.87%), 1-0.9661 (i.e., 3.39%), 1- 0.8576 (i.e., 14.24%) and 1-0.6581(i.e., 34.10%).

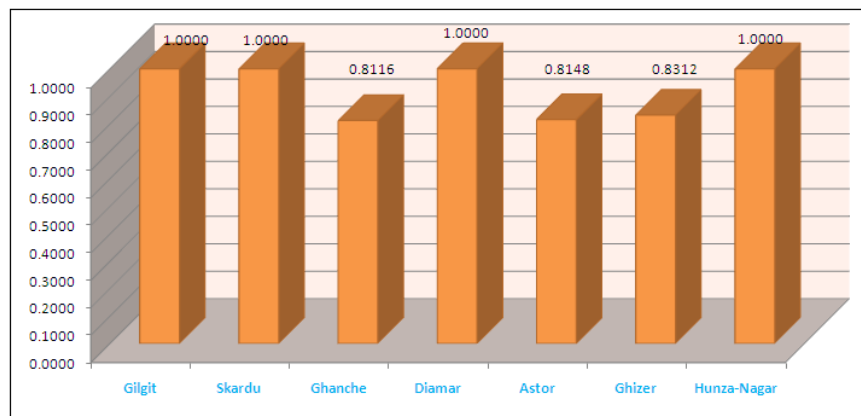


Figure 2 Efficiency Score Chart of 323 Girls district wise schools of Gilgit-Baltistan

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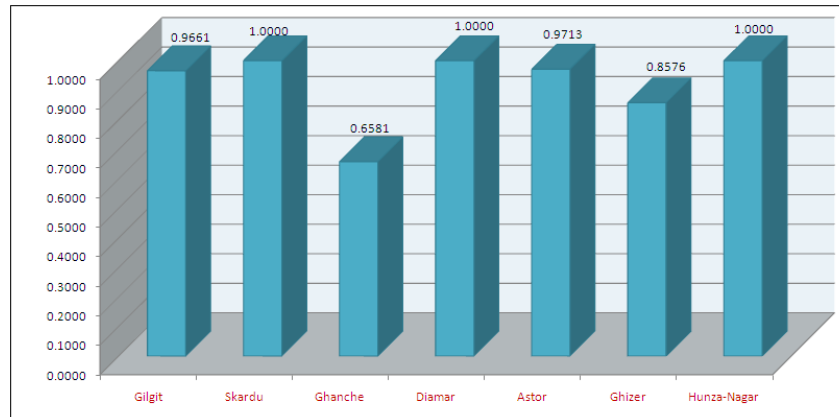


Figure 3 Efficiency Score Chart of 826 Boys district wise Schools of Gilgit-Baltistan

Table: 1 Efficiency measurements using DEA for Girls Schools in Gilgit-Baltistan

DMU's	Efficiency scores	Scale efficiencies	Returns-to-scale	CCR score	NIRS score
<b>Gilgit</b>	1.0000	0.7473	decreasing	0.7473	1.0000
<b>Skardu</b>	1.0000	0.6101	decreasing	0.6101	1.0000
<b>Ghanche</b>	0.8116	0.8473	decreasing	0.6876	0.8116
<b>Diamar</b>	1.0000	1.0000	constant	1.0000	1.0000
<b>Astor</b>	0.8148	0.8171	decreasing	0.6657	0.8148
<b>Ghizer</b>	0.8312	0.8182	decreasing	0.6801	0.8312
<b>HunzaNagar</b>	1.0000	0.8189	decreasing	0.8189	1.0000

Table: 2 Efficiency measurements using DEA for Boys Schools in Gilgit-Baltistan

DMU's	Efficiency scores	Scale efficiencies	Returns-to-scale	CCR score	NIRS score
<b>Gilgit</b>	0.9661	0.9396	increasing	0.9078	0.9078
<b>Skardu</b>	1.0000	0.6753	decreasing	0.6753	1.0000
<b>Ghanche</b>	0.6581	0.9702	increasing	0.6385	0.6385
<b>Diamar</b>	1.0000	1.0000	constant	1.0000	1.0000
<b>Astor</b>	0.9713	0.8387	increasing	0.8147	0.8147
<b>Ghizer</b>	0.8576	0.9538	increasing	0.8179	0.8179
<b>HunzaNagar</b>	1.0000	0.9136	increasing	0.9136	0.9136

Average efficiency in all DMUs of girls' schools is 92.25%, ranging from 81.16 to 100%. According to this result, the share of efficient districts is 57.14% (efficiency score = 100%). As compared to DMUs of boys schools, the average efficiency is 92.18%, ranging from 65.81 to 100%, and the share of efficient districts is 42.85% (efficiency score = 100%) as indicated in Table 3. If we compare girls and boys schools in seven districts at GB, our analysis shows that girl's schools (district) are more efficient, as in Figure 4.

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Table 3: Minimum, maximum and average Efficiency scores and the percentage share of competent schools of districts Gilgit-Baltistan

Parameters	Girls	Boys
Average Score	92.25	92.18
Minimum Score	81.16	65.81
Maximum Score	100	100
Total No. of efficient districts	4	3
Total % share of efficient districts	57.14	42.85

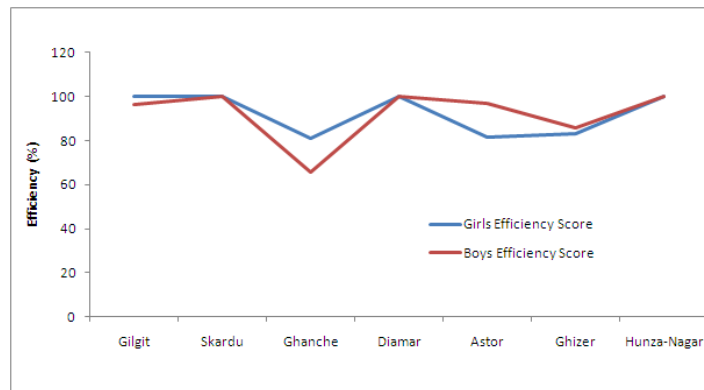


Figure 4: Comparison of Efficiency distribution of Girls and Boys Schools of districts of Gilgit-Baltistan

The above results from DEA model analysis suggest that there is significant efficiency variability among the schools of Gilgit-Baltistan. According to ASER (2021-2023) report [19-21] and other sources the Dimer district has inefficient performance especially in girl's education in ground realities but in our study this district has 100% efficiency score in both boys and girls schools. This may be lack of data or technical mistakes during the survey. According to media reports [20], 85% of children are enrolled in Gilgit-Baltistan as compared to 81% enrolment rate at national level in Pakistan. These results indicated that Gilgit-Baltistan is performing better in the education sector of Pakistan. Whereas, the Diamer district is the lowest performing in terms of enrollment for children aged 5 to 16. This situation is significantly impacting the overall enrollment rate in Gilgit-Baltistan. Furthermore, the data highlights a substantial inequality in enrollment figures between boys and girls in the educational institutions within the Diamer district of Gilgit-Baltistan [20-21]. This paper assessed efficiency for the schools in Gilgit-Baltistan during the study years 2012 and 2013 using DEA. However, the government and education policy makers and other stakeholders could correctly apply DEA approach in its education sector to execute a number of tasks including resources distribution between schools; transferring from the inefficient schools to the proficient schools; identification of "best and poor practices" in school activities; setting target for inefficient schools; continuously monitoring school efficiency changes with time to time, and initiate distribute rewards for good performing schools in Gilgit-Baltistan for best quality of education.

## 4. Conclusion:

It is known that a country's strong education system has optimistic effects not only on its nations but also on its economic growth and development [18]. This paper estimated the efficiency of the schools of seven districts in GB for the year 2012-2013 using DEA. These results can help government and other stake holders to improve on the quality of education, policies, and strategies to increase the outputs of schooling in GB. DEA correlates each DMUs component with all other DMU units, and identifies those units that are working inefficiently in relation to other components definite working result [2].

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Our estimations reveal that four (girls') and three (boy's) schools in the districts of Gilgit-Baltistan sustain 100% efficiency. This indicates that no further upgrading is needed. These results also reveal that a few districts need further development for girls and boys schools in order to reach 100% efficiency. The most inefficient districts that require more improvements are Ghanche, Astor, and Ghizer for girl's schools, and Astor, Gilgit, Ghizer and Ghanche for boy's schools. Among the schools in seven districts of GB, girls' schools are more efficient compared to boy's schools. It should be noted that in ground realities the Diamer district has poor performance in education sector for both boys and girls schooling from known resources [20]. The education sector in Gilgit-Baltistan takes significant potential to serve as a role model for the whole nation. However, it is crucial to implement initiatives at numerous levels and across all districts (currently distributed in 10 districts) to advance the objectives of education.

Finally, this study highlights the strengths and weaknesses in the elementary education sector in Gilgit-Baltistan. However, it is necessary to be careful with the judgments and conclusions that can be drawn from the results. This preliminary research is the first step towards a comprehensive study of the factors that determine the efficiency of educational institutions in Gilgit-Baltistan. The findings obtained from the study area should be communicated to the government administration, public, and executive bodies, as well as the various stakeholders responsible for education in Gilgit-Baltistan. Government should take the necessary actions against the poor performing schools and should introduce rewards for good performing schools in Gilgit-Baltistan in order to increase quality of education.

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