Performance of AICC Statistic as Time Series Modeling and Forecasting of Sunspots

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Abstract: Dynamics of the Sun is quite complex. Sunspot number is an important index to understand the solar dynamo mechanism that governs the solar magnetic cycle. This paper is an attempt to forecast Sunspots number by finding an appropriate time series model. For the purpose, Sunspots datasets from 1749 to 2010 were used. Also, 23 minimum to minimum (m-m) and 24 maximum to minimum (M-m) Sunspots cycles were investigated. The results show that Autoregressive Moving Average (ARMA) time series models fit slightly with the Sunspots number, whereas, Autoregressive Integrated Moving Average (ARIMA) models fit well. They have corrected predictions of the future trend of the Sunspots number within the sample period of study. The results obtained showed that the probability of the AIC corrected model was found better fitted. The problem of over parameterization exited in the model used, but under parameterization was found to minimal.

Key Words: Akaike's Information Corrected Criterion (AICC) Statistic; Sunspots datasets

1. Introduction and Literature Review

Numerous models have been proposed to investigate and forecast the sunspot number by researchers. Among the most popular models are: Final Prediction Error (FPF) paradigm, Schwarz-Ressanen Criteria N (SIC), Bayesian Estimation Criteria N (BEC), Hannan-Cain Criterian, Akaike Information Criteria N (AIC), etc. The more recent model determination criterion is the Akaike Information Corrected Criteria N (AICC) that was created by [10]. Time arrangement displaying is one of the well-known strategies utilized by numerous researchers for estimation [14, 16]. The proposed ARIMA model was used for anticipating climatic conditions and warming relationship. Models could accomplish preferable exactness of load estimate over the conventional ARIMA display [13, 17] proposed an altered ARIMA, which consolidated the administrator's estimation as the underlying anticipating with the temperature and load information in a multi-variable relapse process. The anticipating precision of the changed ARIMA was observed to be superior over conventional ARIMA model. In this study, the strategies of univariate time arrangement investigation are connected to month to month mean information of Sunspots keeping in mind the end goal to assemble models that can used to anticipate the following cyclicity. Time arrangement examination in this investigation has been done utilizing time arrangement programming [9, 11].

The rotation of photospheric outer layer of the Sun cause to appear the dark spots on the outer layer of the Sun called Sunspots [7, 8]. The quantity of Sunspots on the sun based circle is coordinated to be a degree of sun oriented movement. The yearly normal of sunspot regions has been noted beginning around 1700 [2, 15]. The Sunspot Number has been recorded reliably starting around 1755 onwards. Sunspots are the dark spots that appear on the photosphere because of the low temperatures, as compared to the other parts of surface. The thickness of the photosphere is about 400 km and its surface reveal greatest of the solar radiation. The internal and external layer of photosphere has 6,000 K and 4,200 K respectively. Temperature of the sunspots is about 4,600 K. The sunspot is the foremost solar activity; other activities of the sun are associated with sunspots. The average sunspot cycle is 11.1 years, the minimum and maximum cycles of sunspots are 9 and 14 years respectively [3, 18].

Performance of AICC Statistic as Time Series Modeling and Forecasting of Sunspots

The sun oriented cycle depends on the progression of the movement of the sun, the sun based material discharge and the dimension of sunlight based radiations. The presence of sunlight based cycle relies upon the progressions of the numbers of sunspot, flares and in addition to other indicators. To each cycle the most extreme number of sunspots is called as the greatest and least sunspots numbers. The main sun powered cycle is viewed as beginning from 1755, however, the sunlight based cycles have been found first by Samuel Heinrich Schwabe [17, 20]. The present sunspot cycle is 24th cycle.

The maximum sunspot number in cycle 24 is 90, however, it has enormous sunspots as well. Sun oriented greatest existed in May 2014.There were almost no sunspots in 2008 and 2009. The circumstance is exceptionally surprising for nearly a whole century. During 24 sunspot cycle, northern side of the equator has subsequent sunspots of dynamic areas with negative extremity field while southern half of the globe has driving sunspots with positive extremity. It is to be noted that each cycle has distinctive span. Normal length of cycles of sunspot is nearly 11-years.

Every cycle has the number of sunspots, changing as of extreme towards least and rear to greatest once again. The details of corresponding years opposite to cycle 1 to cycle 23 are given as under. Sunspots Cycle-1 consist of (11.3) years, cycle 2 has (9) years, cycle 3 (9.3)years, cycle 4 (13.7) years, cycle 5 (12.6) years, cycle 6 (12.4) years, cycle 7 (10.5) years, cycle 8 (9.8)years, cycle 9 (12.4) years, cycle 10 (11.3) years, cycle 11 (11.8) years, cycle 12 (11.3) years, cycle 13 (11.9) years, cycle 14 (11.5)years, cycle 15 (10) years, cycle 16 (10.1) years, cycle 17 (10.4)years, cycle 18 (10.2)years, cycle 19 (10.5) years, cycle 20 (11.7) years, cycle 21 (10.3) years, cycle 22 (9.7) years, cycle 23 (11.7) years and Sunspots cycle-24 is in continuation started by January 2008 and ending in 2018 and cycle -24 can't analysis completely.

Let us conclude this section by pinpointing the attractive cycles on the solar surface, similar to the sunspot cycles, each attractive cycle is equivalent to the two cycles of sunspots. As far as the base length is concerned, one sunspot cycle is approximately of 11-years, and then this implies that one attractive span of the cycle is 22-years. The peak of attractive cycle, number of sunspots found minimum and maximum that modify two times to each attractive cycle, amid the reversal of polarities, number of sunspots discovered least (Ali et al. 2018).

2. Material and Methodology

This section describes the procedures of establishing appropriate ARIMA model for making forecast. These procedures include: data plotting, data transformation, model selection, parameter estimation, validation tests and forecasting. In this paper, the analysis is carried out using Interactive Time Series Modeling (ITSM). ITSM is totally windows-based computer package for univariate and multivariate time series modeling and forecasting where variance of white noise is the number of observations, and is order of the autoregressive model [4, 9].

This data is then stratified according to the standard 11-years (approximate) solar cycles generating two other datasets.
(i) Monthly mean minimum to minimum (m-m) Sunspot cycles datasets from February 1755 to April 2007 (Cycles: 1-23).
(ii) Monthly mean maximum to minimum (M-m) Sunspot cycles datasets from August 1750 to October 2007 (Cycles: 1-24).

2.1 ARIMA Model

AR model of order p-AR (p)

An autoregressive model denotes present values of the time series in the form of combined one or more preceding values of the same series. It indicates one value dependent on its nearest preceding values. Suppose that if Yt {t = 1, 2, 3 ……. n} is the time series of AR model of order p lacking a constant term, then the expression can be written as:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t \quad \text{... (2.1)}$$

Where e_t error is term and $\phi_1, \phi_2, \ldots, \phi_p$ are autoregressive parameters to be assessed.

Moving Average model of order q- MA (q)

In moving average model on expresses the present values of the time series Yt in linear form of present and preceding values of the white noise series (et). The task of white noise series construction is done on the basis of the forecast errors or residuals when demand observation becomes available. The expression can be written for the moving average model of order q lacking a constant term as:

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \quad \text{... (2.2)}$$

Where e_terror is term and $\theta_1, \theta_2, \ldots, \theta_q$ are parameters to be estimated.

ARMA model of order p, q- ARMA (p, q)

Autoregressive Moving Average model of order p and q is made by joining terms of AR of order p and MA of order q models. Autoregressive Moving Average model of order p and q is generally written as:

$$Y_t = \delta + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \mathrm{e_t} + \theta_1 \mathrm{e_{t-1}} + \theta_2 \mathrm{e_{t-2}} + \cdots + \theta_q \mathrm{e_{t-q}} \quad \ldots.(2.3)$$

The major drawback of ARMA model is that it assumes the time series data as stationary process. On the other hand, the real world data are not stationary in nature. The non-stationary time series data is transformed as stationary by differencing process. Generally, the first order differencing process of time series Yt turn out to be stationary. But, if ARMA time series is transformed as stationary by differencing of order d, it is identified as Integrated Autoregressive Moving Average process and represented by ARIMA (p, d, q).

2.2 ACF and PACF

Both functions the autocorrelation and partial autocorrelation are some kind of graphs that encompass correlations of different time lags. Both ACF and PACF can be used to examine the nature of the series, that whether that is stationary or not and also to classify the number of components in an ARMA model. The number of significant sharp edges in the ACF designates the number of MA terms in the model, while the number of significant edges in PACF shows the number of AR terms in the model.

3. Result and Discussion

The sample ACF and PACF shown in Figs. 1-15 suggest an appropriate ARMA model for the data. The horizontal lines on the graph display the bounds ±1.96 which are approximate 95% bounds for the autocorrelations of the white noise sequence [1,16]. ACF will represent a pure MA (q) model, and the PACF will represent a pure AR (p) model. The estimated models for forecasting the maximum demand of electricity with their corresponding AICC values are given in Tables 1-10. Clearly AR (2) has the minimum AICC.

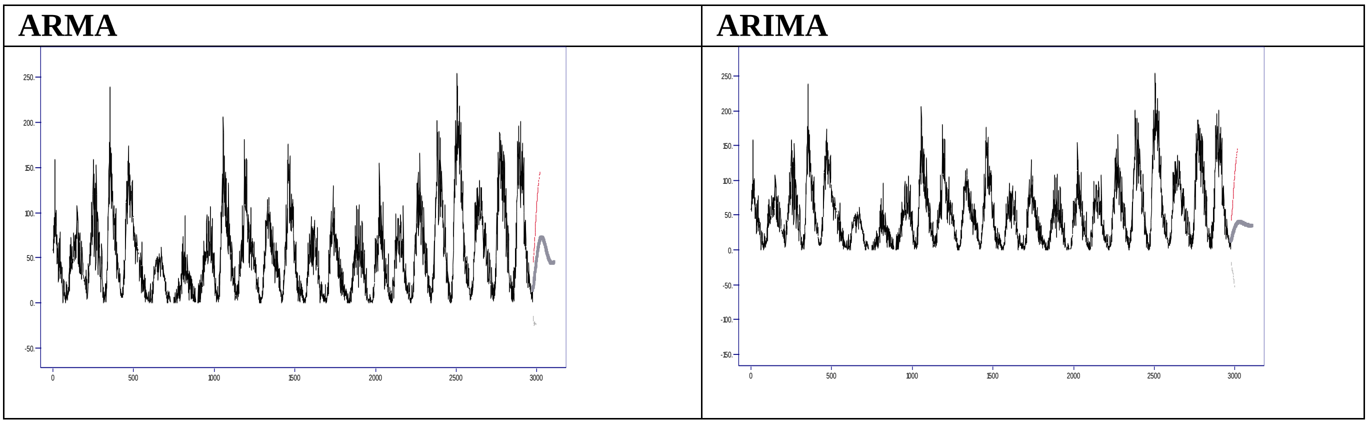


Figure 1 (a–b). SS(2976)1749 to 1996

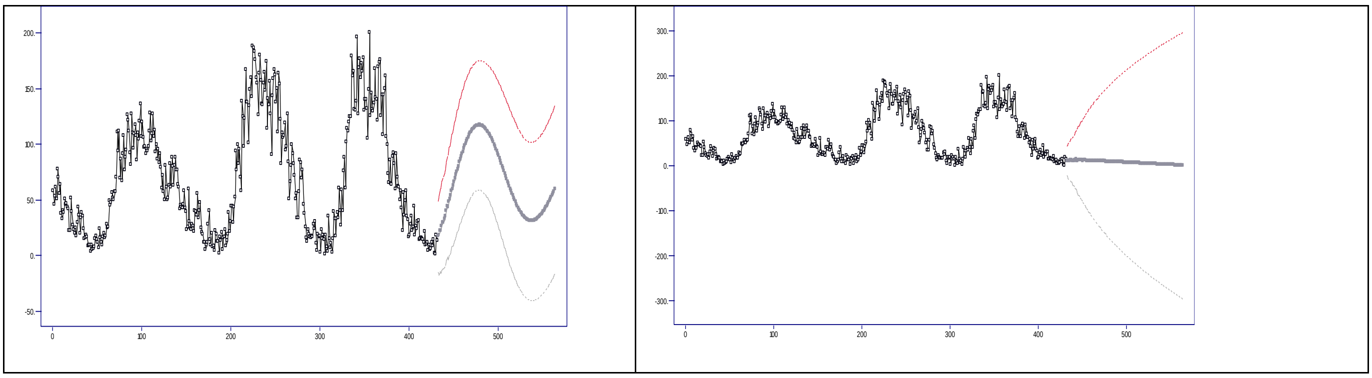


Figure 2 (a–b). SS(432)1961.1 to 1996

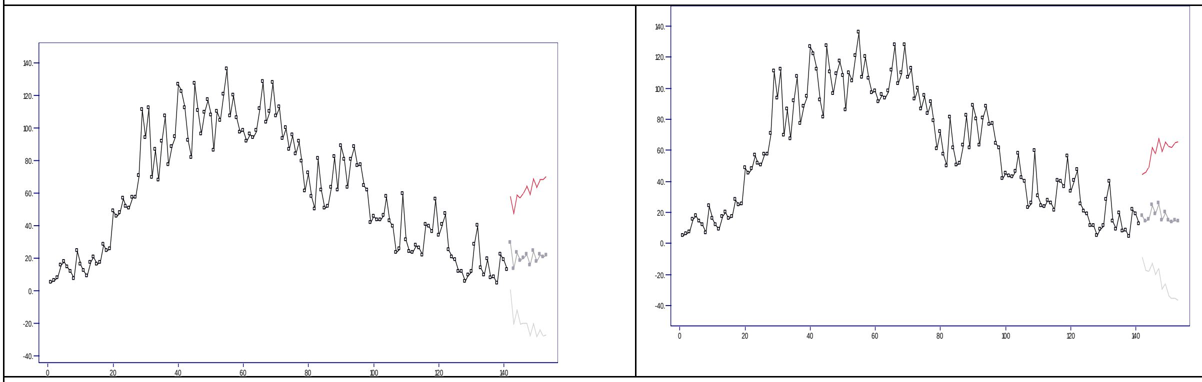


Figure 3 (a–b). SS (m-m) 1964-1976

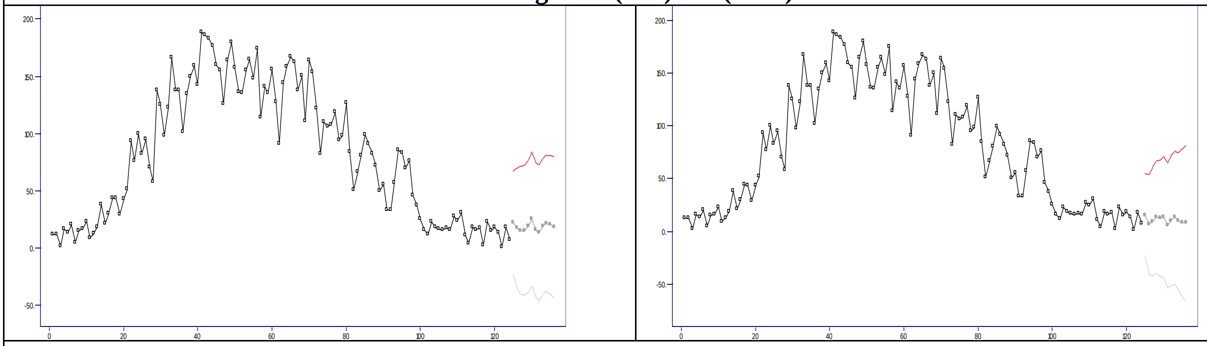


Figure 4 (a–b). SS (m-m) 1976-1986

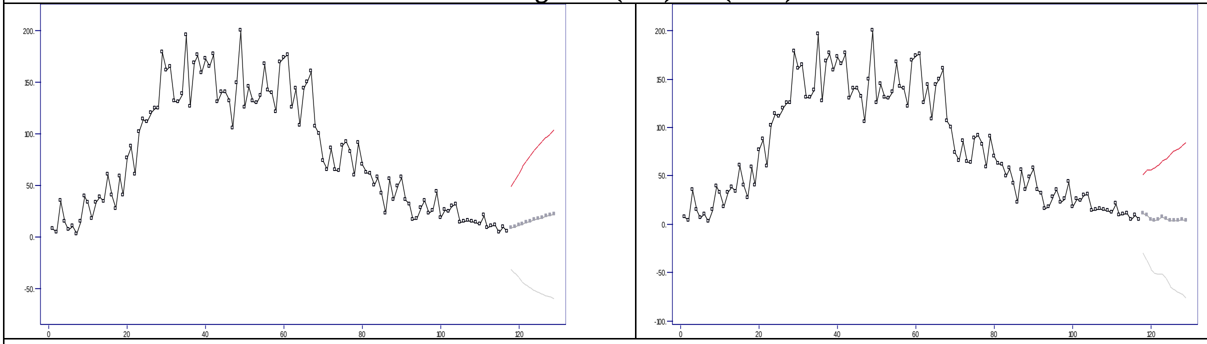


Figure 5 (a–b). SS (m-m) 1986-1996

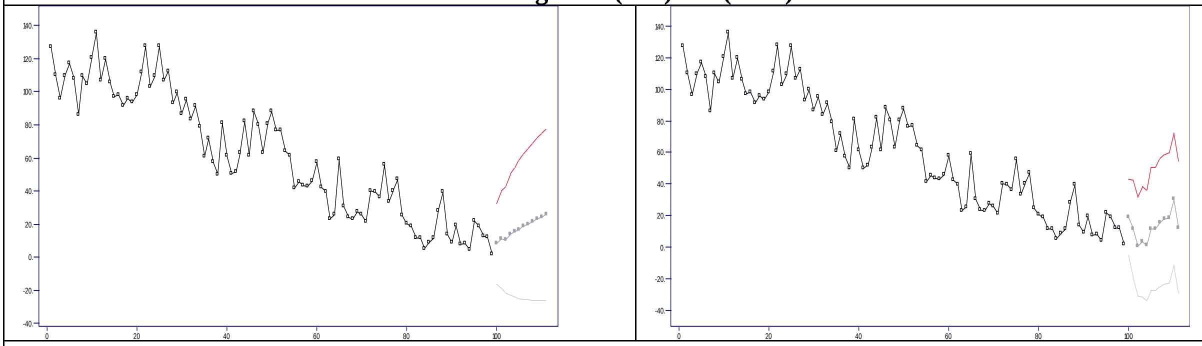


Figure 6 (a–b). SS (M-m) 1968-1976

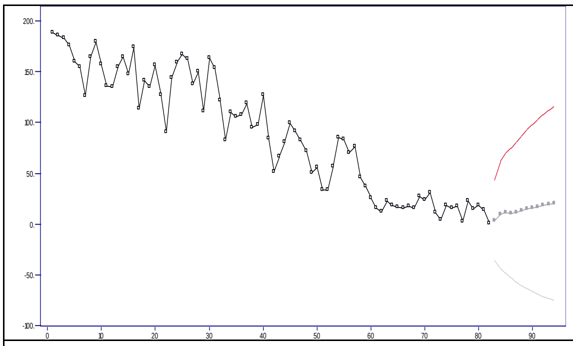
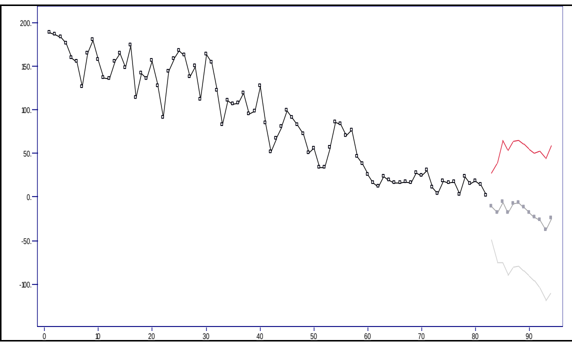



Figure 7 (a–b). SS (M-m) 1979-1986

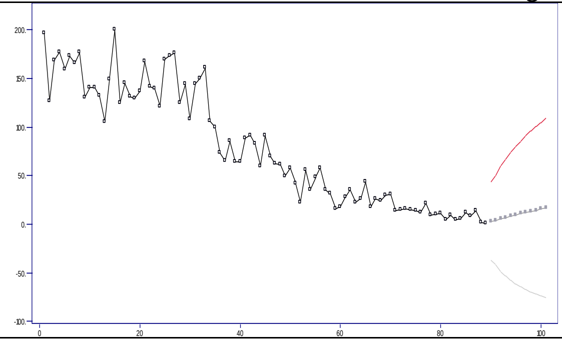
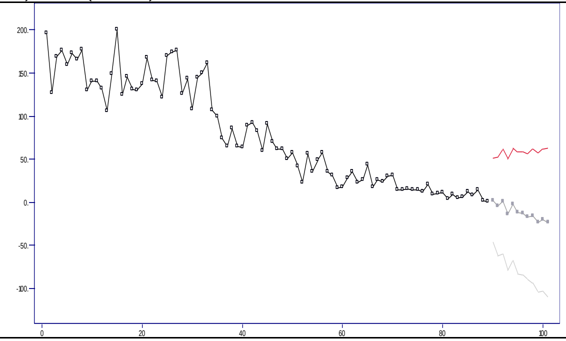



Figure 8 (a–b). SS (M-m) 1989-1996

ARMA ACF/PACF Plot

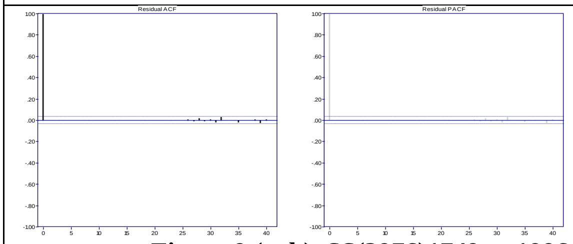
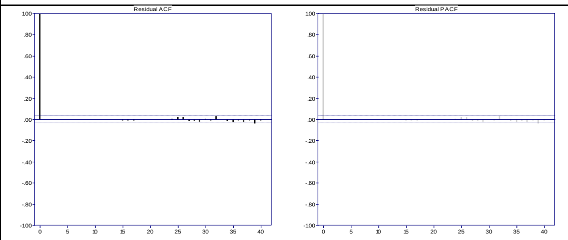
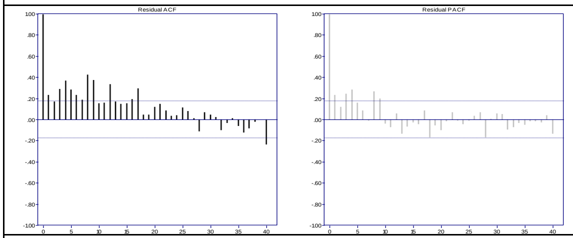


Figure 9 (a–b). SS(2976)1749 to 1996

ARIMA ACF/PACF Plot





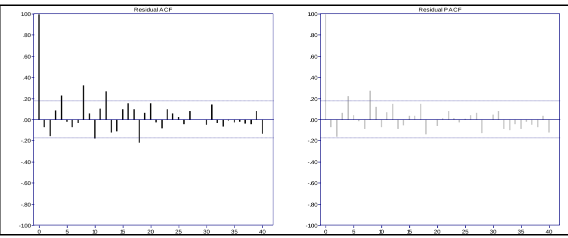
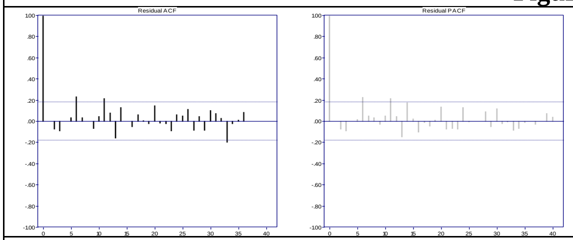


Figure 10 (a–b). SS (m-m)1976-1986



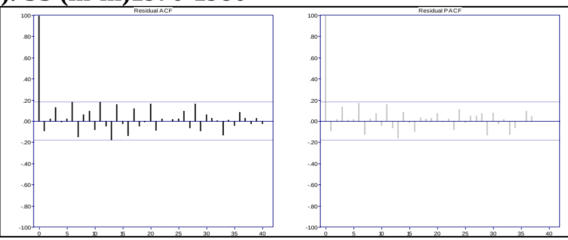
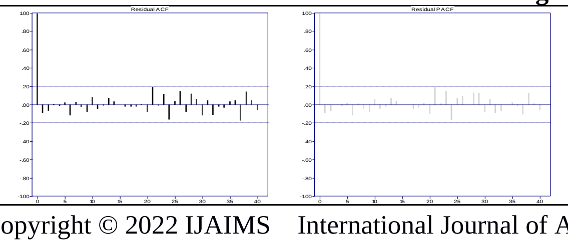


Figure 11(a–b). SS (m-m)1986-1996



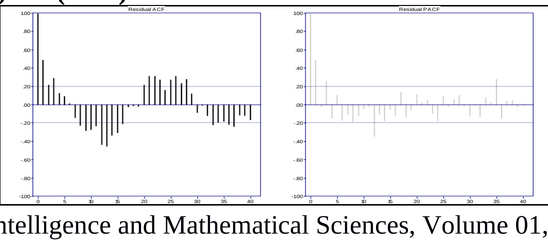


Figure 12 (a–b). SS (M-m) 1968-1976

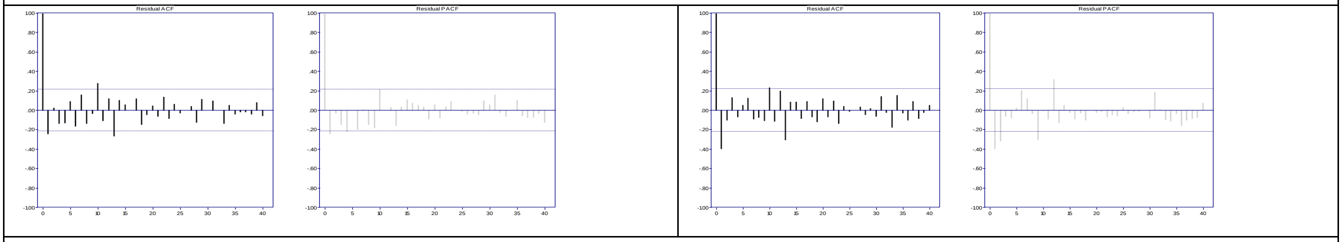


Figure 13 (a–b). SS (M-m)1979-1986

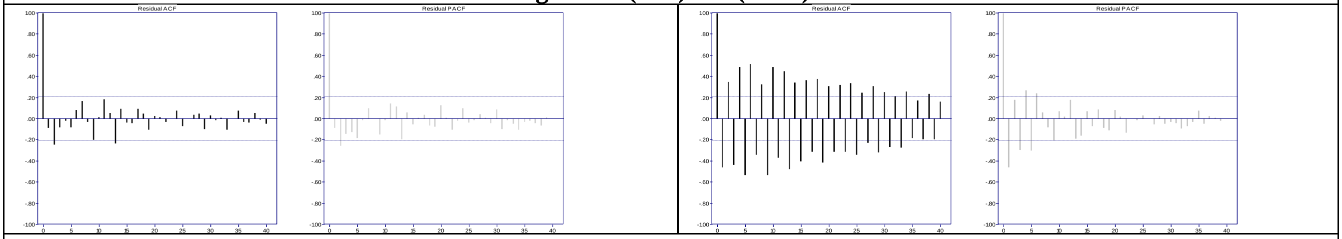


Figure 14 (a–b). SS (M-m)1989-1996

Value can be considered as the most appropriate model if compared among the other models under ARMA. Various classes of time series models, namely ARIMA naiva, Holt's linear and so on are used to forecast the time series.

In the first step we have forecasted the 11-years (1997.1-2007.12) SS datasets. The SS data is first forecasted with the help of actual data from 1749.1 to 1996.12. In the second step we have forecasted the (m-m) cycles (20-23) and (M-m) cycles (21-24) SS datasets falling in the duration (1961.1- 2007.12). We have used the total actual values of a previous cycle to forecast the first twelve months values of the next cycle. Details of ARMA and ARIMA most adequate models, model equations and AICC values are provided in Tables 1-7(a-b) and Figs. 1-7 (a-b). ARMA and ARIMA forecasted versus actual correlation coefficients and the respective p-values are given Tables 1-7(a-b) depict the trends, ARMA and ARIMA models, respective ACF, PACF values and associated graphs 4(a-b)-7(a-b). In case of ARMA model the forecasted values for SS-11 years (1997-2007) obtained with the help of (1749-1997) dataset have a good correlation with the corresponding actual values. However, the ARIMA model is less significant than the ARMA model. ARMA and ARIMA, CCs appear to be 0.706 and 0.422 respectively. The situation improves in case the forecasts are made with the help of (1961-1997) dataset. ARMA and ARIMA CCs appear to be 0.895 and 0.530 respectively.

Table: 1(a). SS Time Series ARMA (1749 to 1996)

ARMA Models	Estimated Model Equations	AICC
(26,0,23)	$X_t = 0.3917\,X_{t-1} + 0.2306\,X_{t-2} - 0.08581\,X_{t-3} - 0.06268\,X_{t-4} + 0.05030\,X_{t-5} + 0.3327\,X_{t-6} + 0.2460\,X_{t-7} + 0.03891\,X_{t-8} + 0.03031\,X_{t-9} + 0.1200\,X_{t-10} - 0.07332\,X_{t-11} + 0.01149\,X_{t-12} - 0.1417\,X_{t-13} - 0.06968\,X_{t-14} + 0.2264\,X_{t-15} - 0.03495\,X_{t-15} - 0.1491\,X_{t-17} - 0.1217\,X_{t-18} + 0.1035\,X_{t-19} + 0.08256\,X_{t-20} - 0.1411\,X_{t-21} + 0.003371\,X_{t-22} - 0.1864\,X_{t-23} + 0.03354\,X_{t-24} + 0.04914\,X_{t-25} + 0.03666\,X_{t-26} + Z_t + 0.1446\,Z_{t-1} - 0.04852\,Z_{t-2} + 0.1556\,Z_{t-3} + 0.2368\,Z_{t-4} + 0.1389\,Z_{t-5} - 0.1565\,Z_{t-6} - 0.2727\,Z_{t-7} - 0.1579\,Z_{t-8} - 0.02788\,Z_{t-9} - 0.1320\,Z_{t-10} - 0.01484\,Z_{t-11} - 0.02562\,Z_{t-12} + 0.08705\,Z_{t-13} + 0.1249\,Z_{t-14} - 0.1332\,Z_{t-15} - 0.09603\,Z_{t-16} + .07699\,Z_{t-17} + .08327\,Z_{t-18} - .08567\,Z_{t-19} - 0.1284\,Z_{t-20} - 0.01302\,Z_{t-21} - 0.04805\,Z_{t-22} + 0.2141\,Z_{t-23}$ where $\{Z_t\} \sim WN(236.250911)$	24815.286929

Table: 2(a). SS(m-m) Time Series ARMA (1964 to 1976)

ARMA Models	Estimated Model Equations	AICC
(3,0,10)	$X_t = 0.2041X_{t-1} + 0.5929X_{t-2} + 0.1908X_{t-3} + Z_t + 0.4300Z_{t-1} - 0.3782\,Z_{t-2} - 0.07435Z_{t-3} - 0.1243\,Z_{t-4} + 0.01669Z_{t-5} - 0.09331Z_{t-6} - 0.03138Z_{t-7} + 0.04667Z_{t-8} - 0.1826Z_{t-9} + 0.1270Z_{t-10}$ where $\{Z_t\} \sim WN(213.982181)$	1193.739070

Table: 3(a). SS(m-m) Time Series ARMA (1976 to 1986)

(4,0,11)	$X_t = 0.7050X_{t-1} - 0.07217X_{t-2} - 0.1121X_{t-3} + 0.4687X_{t-4} + Z_t - 0.1252Z_{t-1}$ $+ 0.09659Z_{t-2} + 0.06831Z_{t-3} - 0.2791Z_{t-4} - 0.1872Z_{t-5} - 0.2072Z_{t-6} + 0.08172Z_{t-7}$ $- 0.2288Z_{t-8} - 0.04518Z_{t-9} + 0.2268Z_{t-10} + 0.05177Z_{t-11}$ where $\{Z_t\} \sim WN(533.357160)$	1171.383180

Table: 4(a). SS(m-m) Time Series ARMA (1986 to 1996)

(3,0,1)	$X_t = 0.5655X_{t-1} + 0.2711X_{t-2} + 0.1335X_{t-3} + Z_t - 0.05750Z_{t-1}$ where $\{Z_t\} \sim WN(428.965710)$	1054.272710

Table: 5(a). SS(M-m) Time Series ARMA (1968 to 1976)

(3,0,1)	$X_t = 0.1249X_{t-1} + 0.4647X_{t-2} + 0.3319X_{t-3} + Z_t + 0.5434Z_{t-1}$ where $\{Z_t\} \sim WN(155.682769)$	793.532585

Table: 6(a). SS(M-m) Time Series ARMA (1979 to 1986)

(4,0,1)	$X_t = 0.4983\, X_{t-1} + 0.2296\, X_{t-2} + 0.04086\, X_{t-3} + 0.2001X_{t-4} + Z_t + 0.3990Z_{t-1}$ where $\{Z_t\} \sim WN(402.473736)$	740.596100

Table: 7(a). SS(M-m) Time Series ARMA (1989 to 1996)

(3,0,1)	$X_t = 1.212X_{t-1} + 0.04646X_{t-2} - 0.2671X_{t-2} + Z_t - 0.6515Z_{t-1}$ where $\{Z_t\} \sim WN(422.100856)$	804.156782

Table: 1(b). SS Time Series ARIMA (1749 to 1996)

ARIMA Models	Estimated Model Equations	AICC
(24,1,5)	$X_t = -0.1196\, X_{t-1} - 0.05951\, X_{t-2} + 0.05319\, X_{t-3} + 0.1187\, X_{t-4} + 0.2818\, X_{t-5}$ $+ 0.1798X_{t-6} + 0.1128\, X_{t-7} + 0.07982\, X_{t-8} + 0.1401\, X_{t-9} + 0.1126\, X_{10}$ $+ 0.08751\, X_{t-11} + 0.08497\, X_{t-12}\ 0.02455\, X_{t-13} + 0.02768\, X_{t-14} + 0.04679\, X_{t-15}$ $- 0.009768\, X_{t-16} + 0.001520\, X_{t-17} - 0.06327\, X_{t-18} - 0.04869\, X_{t-19}$ $- 0.04794\, X_{t-20} - 0.08543\, X_{t-21} - 0.06519\, X_{t-22} + 0.002789X_{t-23} - 0.06961\ _{t-24}$ $+ Z_t - 0.3254\, Z_{t-1} - 0.1282\, Z_{t-2} - 0.1353\, Z_{t-3} - 0.09377\, Z_{t-4} - 0.2322\, Z_{t-5}$ where $\{Z_t\} \sim WN(241.174318)$	24825.827060

Table: 2(b). SS(m-m) Time Series ARIMA (1964 to 1976)

(2,1,15)	$X_t = -0.5463X_{t-1} - 0.06583X_{t-2} + Z_t + 0.1828\, Z_{t-1} - 0.3546Z_{t-2} + 0.06223Z_{t-3}$ $- 0.1002Z_{t-4} + 0.05988Z_{t-5} + 0.008161Z_{t-6} - 0.1032Z_{t-7} + 0.08569Z_{t-8}$ $- 0.2275Z_{t-9} + 0.04485Z_{t-10} - 0.09256Z_{t-11} - 0.06023Z_{t-12} + 0.06612Z_{t-13}$ $+ 0.07256Z_{t-14} + 0.1279Z_{t-15}$ where $\{Z_t\} \sim WN(247.543213)$	1213.505778

Table: 3(b). SS(m-m) Time Series ARIMA (1976 to 1986)

(3,1,10)	$X_t = -0.3870X_{t-1} - 0.3755X_{t-2} - 0.4297X_{t-3} + Z_t + 0.05748Z_{t-1} + 0.1129Z_{t-2}$ $+ 0.04079Z_{t-3} - 0.2393Z_{t-4} - 0.05872Z_{t-5} - 0.1232Z_{t-6} + 0.1282Z_{t-7}$ $- 0.1540Z_{t-8} + 0.08540Z_{t-9} + 0.2915Z_{t-10}$ where $\{Z_t\} \sim WN(445.827693)$	1133.233089

Table: 4(b). SS(m-m) Time Series ARIMA (1986 to 1996)

(2,1,12)	$X_t = -0.7081X_{t-1} - 0.3354X_{t-2} + Z_t + 0.2606Z_{t-1} + 0.002742Z_{t-2} - 0.2817Z_{t-3}$ $- 0.1241Z_{t-4} - 0.06152Z_{t-5} + 0.1115Z_{t-6} + 0.2612Z_{t-7} + 0.04793Z_{t-8} - 0.1778Z_{t-9}$ $- 0.03586Z_{t-10} + 0.1135Z_{t-11} + 0.2128Z_{t-12}$ where $\{Z_t\} \sim WN(398.254378)$	1060.168811

Table: 5(b). SS(M-m) Time Series ARIMA (1968 to 1976)

(2,1,20)	$X_t = -0.3040X_{t-1} - 0.2116X_{t-2} + Z_t + 0.1038Z_{t-1} - 0.4880Z_{t-2} + 0.2395Z_{t-3} - 0.5177Z_{t-4} + 0.5334Z_{t-5} - 0.7542Z_{t-6} + 0.5029Z_{t-7} - 0.2944Z_{t-8} - 0.09833Z_{t-9} + 0.1537Z_{t-10} - 0.1113Z_{t-11} + 0.0006798Z_{t-12} + 0.7846Z_{t-13} - 0.4663Z_{t-14} + 0.5420Z_{t-15} - 0.6929Z_{t-16} + 0.2655Z_{t-17} - 0.5124Z_{t-18} + 0.5597Z_{t-19} - 0.5600Z_{t-20}$ $\text{WN Variance} \sim 113.989146$	891.076666

Table: 6(b). SS(M-m) Time Series ARIMA (1979 to 1986)

(9,1,10)	$X_t = -0.4622X_{t-1} - 0.4830X_{t-2} - 0.3828X_{t-3} - 0.5136X_{t-4} - 0.6211X_{t-5} - 0.3769X_{t-6} + 0.05187X_{t-7} - 0.3221X_{t-8} + 0.1863XX_{t-9} + Z_t + 0.5705Z_{t-1} + 0.4724Z_{t-2} - 0.2085Z_{t-3} - 0.01459Z_{t-4} + 0.05220Z_{t-5} - 0.2324Z_{t-6} - 0.3053Z_{t-7} - 0.06297Z_{t-8} - 0.09015Z_{t-9} + 0.0002028Z_{t-10}$ $\text{where} \{Z_t\} \sim WN(467.754102)$	789.219133

Table: 7(b). SS(M-m) Time Series ARIMA (1989 to 1996)

(5,1,12)	$X_t = -0.4489X_{t-1} - 0.6864X_{t-2} - 0.2652X_{t-3} - 0.4053X_{t-4} + 0.06488X_{t-5} + Z_t + 0.07377Z_{t-1} + 0.3132Z_{t-2} - 0.06392Z_{t-3} - 0.1517Z_{t-1} + 0.04320Z_{t-5} - 0.4142Z_{t-6} + 0.2310Z_{t-7} - 0.03393Z_{t-8} + 0.2503Z_{t-9} - 0.1978Z_{t-10} + 0.2341Z_{t-11} - 0.2194Z_{t-12}$ $\text{where} \{Z_t\} \sim WN(576.017550)$	865.770969

Table 8: CC of 11 years Forecasts vs. Actual Datasets

Range of Forecasts	ARMA		ARIMA	
	CC	***P*-Value**	CC	***P*-Value**
SS 11 years (1997.1-2007.12) forecasts using (1749.1-1997.12) dataset	0.706	0.000	0.422	0.000
SS 11 years (1997.1-2007.12) forecasts using SS (1961.1-1997.12)	0.895	0.000	0.530	0.000

Table 9: CC of Forecasted vs. Actual Datasets (m-m) SS

Range of Forecasts	ARMA		ARIMA	
	CC	***P*-Value**	CC	***P*-Value**
SS cycle 21, 12 months forecasts for 1976.6-1977.5 using all actual values of cycle 20 (1964.9-1976.5)	-0.170	0.597	-0.040	0.903
SS cycle 22, 12 months forecasts for 1986.9-1987.8 using all actual values of cycle 21 (1976.5-1986.8)	-0.439	0.153	-0.554	0.062
SS cycle 23, 12 months forecasts for 1996.5-1997.4 using all actual values of cycle 22 (1986.8-1996.4)	0.198	0.537	-0.238	0.456

Table 10: CC of Forecasted vs. Actual Datasets (M-m) SS

Performance of AICC Statistic as Time Series Modeling and Forecasting of Sunspots

Range of Forecasts	ARMA		ARIMA	
	CC	*P*-Value	CC	*P*-Value
SS cycle 22, 12 months forecasts for 1976.8-1977.7 using all actual values of cycle 21 (1968.5-1976.7)	0.272	0.393	0.547	0.066
SS cycle 23, 12 months forecasts for 1986.7-1987.6 using all actual values of cycle 22 (1979.9-1986.6)	0.253	0.428	-0.621	0.031
SS cycle 24, 12 months forecasts for 1996.11-1997.10 using all actual values of cycle 23 (1989.6-1996.10)	0.568	0.054	-0.519	0.084

Details are depicted in Tables 1-10. The spectral estimation using wavelets (Cowan 2007) also confirms the above results. The associated period grams are depicted in the above figure. Maximum values of the means follow these cyclicities with slight differences.

4. Conclusion

As already discussed the (m-m), and (M-m) cycles have significant correlations. SS cycle 21 (first 12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be -0.170 and -0.040 respectively. ARMA and ARIMA forecasts are not significantly correlated with the actual values. SS cycle 22 (first 12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be -0.439 and -0.554 respectively. ARIMA forecasts are significantly correlated with the actual values whereas the ARMA forecasts are poorly correlated with the actual values. SS cycle 23 (first12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be 0.198 and -0.238 respectively. Both the ARMA and ARIMA forecasts are not significantly correlated with the actual values. Details are depicted in Table 8.

SS cycle 22 (first 12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be 0.272 and 0.547 respectively. ARIMA forecasts are significantly correlated with the actual values whereas the ARMA forecasts are poorly correlated with the actual values. SS cycle 23 (first 12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be 0.253 and -0.621 respectively. ARIMA forecasts are significantly correlated with the actual values whereas the ARMA forecasts are poorly Correlated with the actual values. SS cycle 24 (first 12 months) ARMA and ARIMA CCs for forecasted versus actual values appear to be 0.568 and -0.519 respectively. Both the ARMA and ARIMA forecasts are significantly correlated with the actual values. Details are depicted in Tables 1-7 (a-b).

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